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Gas damping force noise on a macroscopic test body in an infinite gas reservoir

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ABSTRACT

We analyze here the force noise associated with the mechanical damping of a test body surrounded by a large volume of rarefied gas, considering the statistics of momentum exchange in inelastic molecular collisions. In addition to calculating the force noise on a cube, sphere, and cylinder, we discuss the limits in applying this analysis to experimental configurations in which the test body is surrounded by an enclosure that is only slightly larger than the test body itself.

1. Introduction

Gas damping of the motion of a macroscopic test body is a potentially sensitivity-limiting source of Brownian noise in variety of experiments that are sensitive to very small forces. Gas damping in the molecular flow regime, with mean free paths longer than the test body itself, is characterized by a viscous damping coefficient $\beta_{tr} = -\frac{dF}{dV}$ in translation or $\beta_{rot} = -\frac{dN}{d\dot{\phi}}$ in rotation, and has been found in numerous torsion pendulum experiments to be proportional to the residual gas pressure p [1–3] (here, F and N represent, respectively, force and torque, and V and $\dot{\phi}$ are translational and angular velocities). The power spectrum of Brownian force noise associated with the molecular impacts is related to the damping coefficient via the fluctuation–dissipation theorem, which gives $S_F = 4k_B T\beta_{tr}$ (T is the system temperature and k_B is the Boltzmann constant). A quantitative study of the gas damping and consequent force noise acting on small levitated spheres was performed by Hinkle and Kendall [4,5] and experimentally verified the Brownian character of residual gas force noise.

The order of magnitude of the translational motion gas damping coefficient can be found easily as follows: a test body – or test mass, referred to here as TM – with section A that is moving with velocity of magnitude V in a gas with molecular number density n (related to pressure and temperature by $n = \frac{p}{k_B T}$) will be struck, on average, by order nAV more molecules per second on the "upwind" side than on the "downwind" side. Each molecule imparts a momentum of order $m_0 v_T$ where m_0 is the mass of the molecule and $v_T \equiv \sqrt{\frac{k_B T}{m_0}}$ is the characteristic thermal velocity. This gives a translational damping coefficient $\beta_{tr} \approx nA \times m_0 v_T \approx \frac{pA}{v_T}$.

This general dependence is found, albeit with different prefactors, in the analysis of gas damping force noise from damping of mirror vibrational modes for gravitational wave interferometers and other oscillators [6–8], and for the force noise on the cubic TM for a space gravitational wave interferometer [9]. To get an exact number, we must consider not only the momentum exchange normal to the surface, but also the role of shear forces acting parallel to the TM surface. Additionally, we must consider the test mass recoil from molecules leaving the surface, which is a process that is correlated, via the conservation of particles, with the incoming collisions.

This Letter presents a calculation of the gas damping coefficient, with particular attention to the cubic TM geometry. This has been motivated by the need for an accurate modeling of gas damping noise on the cubic TM that serve as geodetic reference masses in the Laser Interferometry Space Antenna (LISA [10]) and LISA Pathfinder (LPF [11]) missions, in which the TM must be free of spurious accelerations at the level of $\mathrm{fm/s^2/Hz^{1/2}}$. The dissipation analysis for a cube is demonstrated first with a force noise evaluation and then confirmed with direct calculation of the damping coefficient. The force noise analysis is also extended to other simple geometries for both translation and rotation.

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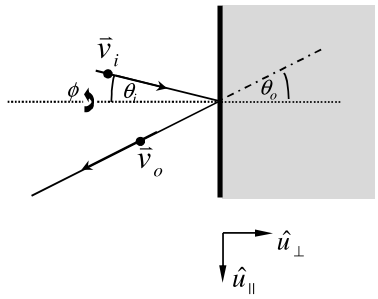


Fig. 1. Illustration of inelastic molecular collision with a test body surface, with incoming velocity $\vec{v}_i$ and subsequent reemission with outgoing velocity $\vec{v}_o$. Random, diffuse scattering, reemission with a $\cos\theta_o$ distribution is assumed in the calculation.

The calculation presented is straightforward and based on the simple physics of diffuse scattering, used, for instance, in estimating molecular flow conductances in tubes. It is of current value, however, for several reasons. First, a quantitative calculation of gas damping is important for the design of gravitational wave experiments, and previous estimates appear to have underestimated this effect by roughly an order of magnitude in noise power [9]. The model presented is also easily extended to, and can be tested by, torsion pendulum small force experiments. Finally, and most importantly, the limitations of the applicability of this calculation – namely the assumption of having a TM surrounded by an infinite gas volume, which is discussed in the conclusion – indicate that the force noise from gas damping can be grossly underestimated in many experiments, including LISA, which is the subject of a more detailed experimental and numerical study [12]. The accurate calculation of damping in an infinite gas volume is thus the necessary conceptual starting point, as well as an analytical anchor point, for numerical or experimental studies of damping in tighter geometries.

2. Calculation

2.1. Force noise calculation

2.1.1. Force noise normal and parallel to a surface element

We can calculate the single-sided power spectrum of the force noise per unit area – which we define $S_\perp$ and $S_\parallel$ for components normal and parallel to the surface – from the mean square fluctuations of the time average force exerted in a time interval T_0. For uncorrelated molecular impacts, which we treat as delta-functions in time, the time average force on a surface element ΔA can be written

$$\bar{F}_\perp \equiv \frac{1}{T_0}\int_0^{T_0} dt F_\perp(t) = \frac{1}{T_0}\sum_k m_0(\vec{v}_{ik} - \vec{v}_{ok}).\hat{u}_\perp \tag{1}$$

Here, $\vec{v}_{ik}$ and $\vec{v}_{ok}$ are the incoming and outgoing velocities of the molecule involved in collision k. An analogous expression holds for the force components parallel to the surface ($\hat{u}_\perp$ and $\hat{u}_\parallel$ represent unit vectors perpendicular and parallel to the TM surface element).

For white force noise, as would be expected for momentum transfer from uncorrelated, delta-function collisions of single molecules against a surface in the regime of molecular flow, the single-side noise spectrum can be expressed

$$S_\perp \Delta A = 2T_0\left\langle\left(\bar{F}_\perp - \langle\bar{F}_\perp\rangle\right)^2\right\rangle \tag{2}$$

with an analogous expression for $S_\parallel$, the force noise per unit area parallel to a surface element.[1]

The calculation can be simplified by considering a surface element small enough – and time interval short enough – such that the probability of more than one collision becomes negligible. In this limit – $n v_T T_0 \Delta A \ll 1$ – the calculations of the expectation value of the average force and its mean square become simple integrals over the probability, $P(\vec{v}_i, \vec{v}_o)$, of a single collision occurring, in the interval T_0, with incoming and outgoing velocities $\vec{v}_i$ and $\vec{v}_o$ (see Fig. 1),

$$\langle\bar{F}_\perp\rangle = \frac{m_0}{T_0}\int d^3\vec{v}_i\, d^3\vec{v}_o\, P(\vec{v}_i, \vec{v}_o)(\vec{v}_i - \vec{v}_o).\hat{u}_\perp \tag{5}$$

$$\left\langle\bar{F}_\perp^2\right\rangle = \frac{m_0^2}{T_0^2}\int d^3\vec{v}_i\, d^3\vec{v}_o\, P(\vec{v}_i, \vec{v}_o)\left[(\vec{v}_i - \vec{v}_o).\hat{u}_\perp\right]^2 \tag{6}$$

The probability $P(\vec{v}_i, \vec{v}_o)$ can be expressed

$$\begin{aligned} &P(\vec{v}_i, \vec{v}_o)\, d^3\vec{v}_i\, d^3\vec{v}_o \\ &\quad = n\left(\frac{1}{2\pi v_T^2}\right)^{3/2} \exp -\frac{v_i^2}{2v_T^2}(\Delta A\cos\theta_i v_i T_0)\left(v_i^2\, dv_i\, d\Omega_i\right) \\ &\qquad \times \frac{1}{2\pi v_T^4}\cos\theta_o v_o \exp -\frac{v_o^2}{2v_T^2}\left(v_o^2\, dv_o\, d\Omega_o\right) \end{aligned} \tag{7}$$

where the velocities are expressed with magnitudes v and solid angles Ω (angles ϕ and θ). The first term in Eq. (7) is the probability of having a molecule, assumed to obey a Maxwell–Boltzmann distribution with number density n, in the tube of volume $dA\cos\theta_i v_i T_0$ that will hit the wall element in the next time interval T_0 with velocity $\vec{v}_i$. The second term corresponds to the probability, given the arrival of a molecule, of reemission with speed v_o into solid angle Ω_o. The impacts are assumed to be completely inelastic and "memory erasing", with the outgoing angle independent of the incoming angle, and reemission is assumed to occur immediately after the impact. Weighting by the factor $v_o\cos\theta_o$, which ensures that the incoming and outgoing distributions are identical, is the cosine law for diffuse scattering, which is used, with substantial experimental verification, for estimating Knudsen diffusion in pipes and other geometries (see, for instance, Refs. [13–15]). We will discuss the impact of other assumptions of the molecule – TM interaction at the conclusion of the calculation.

We can calculate the expected average force, with $(\vec{v}_i - \vec{v}_o).\hat{u}_\perp = v_i\cos\theta_i + v_o\cos\theta_o$ and restriction of the integral over

[1] The single-sided power spectrum $S_{xx}(\omega)$ for a stochastic process $x(t)$, assumed here to be zero mean and stationary – such as for $x = F_\perp - \langle F_\perp\rangle$ in Eq. (2) – is related to the autocorrelation function, $C_{xx}(\tau) \equiv \langle x(t)x(t+\tau)\rangle$ via

$$S(\omega) = 2\int_{-\infty}^{\infty} C_{xx}(\tau)\exp -i\omega\tau\, d\tau \tag{3}$$

White noise, with a frequency independent power spectrum S_0, thus corresponds to an autocorrelation function $C_{xx}(\tau) = \frac{S_0}{2}\delta(\tau)$. With $\bar{x} \equiv \frac{1}{T_0}\int_0^{T_0} dt\, x(t)$, we can evaluate the expectation value $\langle\bar{x}^2\rangle$ and find

$$\begin{aligned} \left\langle\bar{x}^2\right\rangle &= \frac{1}{T_0^2}\int_0^{T_0} dt_1 \int_0^{T_0} dt_2 \left\langle x(t_1)x(t_2)\right\rangle \\ &= \frac{1}{T_0^2}\int_0^{T_0} dt_1 \int_0^{T_0} dt_2\, \frac{S_0}{2}\delta(t_2 - t_1) \\ &= \frac{S_0}{2T_0} \end{aligned} \tag{4}$$

This reasoning leads to Eq. (2) and to the final simplification before Eq. (10).

the incoming and outgoing velocities to the positive hemisphere (θ_i, θ_o in the range $[0, \pi/2]$):

$$\langle \bar{F}_\perp \rangle = m_0 \Delta A n v_T^2 = p \Delta A \tag{8}$$

with the final obvious result emerging with the ideal gas law, $p = nk_BT$.

In similar fashion we can integrate to obtain the mean square average force,

$$\langle \bar{F}_\perp^2 \rangle = \frac{\Delta A}{T_0} p \left(\frac{8 m_0 k_B T}{\pi} \right)^{1/2} \left(1 + \frac{\pi}{4} \right) \tag{9}$$

Inspection of Eqs. (8) and (9) shows that $\langle \bar{F}_\perp^2 \rangle \gg \langle \bar{F}_\perp \rangle^2$ in the studied single-collision regime, and so Eq. (2) simplifies to $S_\perp = \frac{2T_0}{\Delta A} \langle \bar{F}_\perp^2 \rangle$, and thus

$$S_\perp = p \left(\frac{32 m_0 k_B T}{\pi} \right)^{1/2} \left(1 + \frac{\pi}{4} \right) \tag{10}$$

We note that the "extra" contribution $\frac{\pi}{4}$ comes from the cross term $\langle v_i \cos\theta_i v_o \cos\theta_o \rangle$, which can be thought of as a correlation between the momentum imparted normal to the surface, which is always positive, by the arrival and subsequent reemission of a molecule. The remaining term – unity in parentheses in Eq. (10) – comes from equal contributions of the incoming and outgoing molecules.

If, in alternative, we were to consider the incoming and outgoing particle fluxes as uncorrelated processes, which would be true for molecules that stick to the surface for a long duration – much longer than the reciprocal of the frequency where we consider the force noise – before reemission, the contribution $\frac{\pi}{4}$ disappears. Another interesting case is that of elastic collisions with specular reflection, which is equivalent to replacing the outgoing distribution with a delta function for the specular reflection of the incoming velocity, the factor $(1 + \frac{\pi}{4})$ is replaced by a factor 2.

We can perform a similar calculation for either of the force components parallel to the surface. For this we replace, in evaluating Eq. (6), the momentum exchange normal to the surface with one of the components parallel to the surface,[2]

$$m_0 (\vec{v}_i - \vec{v}_o).\hat{u}_\parallel = v_i \sin\theta_i \cos\phi_i - v_o \sin\theta_o \cos\phi_o \tag{11}$$

For this parallel component, the expectation average force is zero. Also, in the mean square of the average force, there is no effect of correlation between the incoming and outgoing molecules, as the azimuth angles are independent of one another and both have zero mean momentum exchange parallel to the surface. We thus obtain

$$S_\parallel = p \left(\frac{8 m_0 k_B T}{\pi} \right)^{1/2} \tag{12}$$

Considering the alternative cases discussed above, there is no difference in the case of sticking molecules, as the incoming and outgoing processes are uncorrelated. In the case of specular reflection, the velocity parallel to the surface element is conserved and thus $S_\parallel = 0$.

Finally, we note that in the diffuse scattering hypothesis there is no cross-correlation between the different components of the force noise, given the azimuthally random reemission of molecules.

2.1.2. Total force and torque noise on a cubic TM and other geometries

The force noise normal Eq. (10) and parallel Eq. (12) to a unit surface element can be integrated over the surface of a test body to give the total force along a chosen axis. For a cubic test mass, this calculation is particularly simple, with the relevant force noise component per area (S_z for the z axis) uniform on each face, given by $S_\perp$ on the two faces (Z) normal to z and by $S_\parallel$ on the four faces (X and Y) parallel to z. For a cube of side length s, we obtain a total force noise

$$\begin{aligned} S_F &= 2 \int_Z dx\,dy\, S_\perp + 2 \int_X dy\,dz\, S_\parallel + 2 \int_Y dx\,dz\, S_\parallel \\ &= p s^2 \left(\frac{512 m_0 k_B T}{\pi} \right)^{1/2} \left(1 + \frac{\pi}{8} \right) \end{aligned} \tag{13}$$

We can also evaluate the torque noise along a chosen axis, which is of relevance to torsion pendulum experiments. Considering the torque along the z axis, $N_z = xF_y - yF_x$, we integrate the torque noise per area for every surface element, $S_{N_z} = x^2 S_y + y^2 S_x$, over the faces of the cube. For the Z faces, only shear forces, in x and y contribute to the torque. The four lateral faces each have contributions from both shear, with armlength $\frac{s}{2}$, and normal forces, with an armlength that varies across the cubic face. Integrating, we obtain

$$\begin{aligned} S_N &= 2 \int_X dy\,dz \left[y^2 S_\perp + \left(\frac{s}{2} \right)^2 S_\parallel \right] \\ &\quad + 2 \int_Y dx\,dz \left[x^2 S_\perp + \left(\frac{s}{2} \right)^2 S_\parallel \right] \\ &\quad + 2 \int_Z dx\,dy \left[x^2 S_\parallel + y^2 S_\parallel \right] \\ &= \frac{s^4}{3} S_\perp + \frac{4 s^4}{3} S_\parallel \\ &= p s^4 \left(\frac{32 m_0 k_B T}{\pi} \right)^{1/2} \left(1 + \frac{\pi}{12} \right) \end{aligned} \tag{14}$$

The calculation of the total force and torque noise has also been performed for a sphere and a right cylinder, by straightforward integration of Eqs. (10) and (12) over the TM surface (results in Table 1). We note that the rotational damping (or, equivalently, the torque noise) on a sphere or cylinder, the latter along its axis of symmetry, is due entirely to the shear forces and thus would be zero in a model with elastic molecular collisions.

We note that the results of Table 1 have been confirmed to roughly the percent level, for these three geometries and in both translation and rotation, by a numerical simulation that traces the impulses of a single molecule as it impacts the test mass inside a much larger volume, with the reemission distribution governed by the diffuse scattering statistics of Eq. (7).[3] The simulation technique is described in Ref. [12].

2.2. Direct calculation of viscous damping coefficient for a cube

As a simple crosscheck for the force noise calculated for a cube in the previous section, we can directly calculate the viscous damping coefficient β by calculating the force on a cubic test body moving with velocity $\vec{V} = V_\perp \hat{u}_\perp$ with respect to the surrounding gas. Damping will occur both due to the molecules which hit the test mass on all faces – with a velocity distribution that becomes

[2] The resulting noise is the same for the other parallel component, with $\sin\phi$ replacing $\cos\phi$ in the calculation.

[3] We note that our analytic results, and their confirmation by numerical simulation, present a slight discrepancy compared to the result of [5] (Eq. (6) in that article) for the translational gas damping of a sphere, in the diffuse scattering hypothesis and apparently including the correlation between incoming and outgoing molecules. That result has a term $\frac{\pi}{16}$ in place of the $\frac{\pi}{8}$ given in Table 1 here.

asymmetric, in the test mass reference frame, due to $V_\perp$ – and due to the reemitted molecules – which, though emitted with the same velocity distribution on all sides of the test mass, still produce a net force, as more molecules will be emitted from the upwind side.

The expectation average force on a surface element on the upwind or downwind faces – the faces perpendicular to the test mass motion – can be calculated, as in Eq. (5), by integrating the momentum exchange $m_0(v_{i\perp} - v_{o\perp})$ over the joint probability function, $P_{V_\perp}$, which assumes an asymmetry in the reference frame of the moving test mass,

$$\begin{aligned} &P_{V_\perp}(\vec{v}_i, \vec{v}_o)\, d^3\vec{v}_i\, d^3\vec{v}_o \\ &\quad = n\left(\frac{1}{2\pi v_T^2}\right)^{3/2} \exp - \frac{(v_{i\perp} + V_\perp)^2 + v_{i\parallel}^2}{2v_T^2} \\ &\qquad \times (\Delta A v_{i\perp} T_0)\left(v_i^2\, dv_i\, d\Omega_i\right) \\ &\qquad \times \frac{1}{2\pi v_T^4} v_{o\perp} \exp - \frac{v_o^2}{2v_T^2}\left(v_o^2\, dv_o\, d\Omega_o\right) \end{aligned} \tag{15}$$

The integrals can be simplified in the limit of $V_\perp \ll v_T$ and neglecting terms of order $V_\perp^2$, to yield, respectively, for the downwind and upwind faces,

$$\langle \bar{F}_\perp \rangle = p\Delta A\left[\pm 1 - V_\perp\left(\frac{2m_0}{\pi k_B T}\right)^{1/2}\left(1 + \frac{\pi}{4}\right)\right] \tag{16}$$

Again, had we assumed elastic collisions, which essentially multiplies the force of the incoming molecules by two, the factor $(1 + \frac{\pi}{4})$ would become 2, which would produce the same result obtained by Christian [8] for the difference in pressure on two sides of a membrane.

We can similarly calculate the expected force component for a surface element on the faces parallel to the TM motion, following Eq. (11), which yields

$$\langle \bar{F}_\parallel \rangle = -p\Delta A V_\parallel \left(\frac{m_0}{2\pi k_B T}\right)^{1/2} \tag{17}$$

Eqs. (16) and (17) can be integrated – actually a simple multiplication by the surface area s^2 – over the six faces of the cube. The leading term in Eq. (16) cancels out upon summing the upwind and downwind faces, to yield a pure velocity damping force,

$$\beta \equiv -\frac{dF_x}{dV_x} = ps^2\left(\frac{32m_0}{\pi k_B T}\right)^{1/2}\left(1 + \frac{\pi}{8}\right) \tag{18}$$

This agrees with the value of β, given in Table 1 and calculated from the force fluctuations and thus, as expected, agrees with the fluctuation–dissipation theorem.

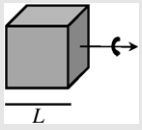
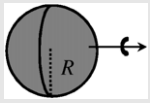
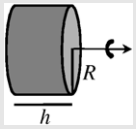
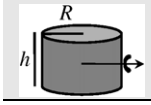
3. Discussion

The expressions derived here for the gas damping on a cubic test mass are thus confirmed by two independent calculations. Both calculations of course rest on the assumptions of diffuse scattering of molecules along the test mass surface. This would be fairly easy to test, for the cube (or similar plate) and cylinder geometries for the pressure dependence of the rotational damping coefficient in a torsion pendulum. We also note that measuring the rotational damping for a cylinder is a direct test of the diffuse scattering hypothesis, as elastic scattering would give no rotational damping.

Applying Eq. (13) to a 46 mm cubic TM, as currently employed for LISA, the expected force noise from molecular impacts in an infinite gas volume would be

$$S_F^{1/2} = 0.75\ \mathrm{fN/Hz^{1/2}}\left(\frac{p}{1\ \mu\mathrm{Pa}}\right)^{1/2}\left(\frac{s}{46\ \mathrm{mm}}\right)^2\left(\frac{m_0}{30m_p}\right)^{1/4} \tag{19}$$

Table 1
Calculated translational and rotational damping coefficients for different test bodies with cubic, spherical, and cylindrical shapes, along the axes indicated in the figures at left. Note that the damping coefficients are related to the force and torque noise spectra via the fluctuation–dissipation formulas, $S_F = 4k_B T\beta_{tr}$ and $S_N = 4k_B T\beta_{rot}$.

Shape	$\beta_{tr}/[p/v_T]$	$\beta_{rot}/[p/v_T]$
	$s^2(\frac{32}{\pi})^{1/2}(1+\frac{\pi}{8})$	$s^4(\frac{2}{\pi})^{1/2}(1+\frac{\pi}{12})$
	$\pi R^2(\frac{128}{9\pi})^{1/2}(1+\frac{\pi}{8})$	$\pi R^4(\frac{32}{9\pi})^{1/2}$
	$\pi R^2(\frac{8}{\pi})^{1/2}(1+\frac{h}{2R}+\frac{\pi}{4})$	$\pi R^4(\frac{1}{2\pi})^{1/2}(1+\frac{2h}{R})$
	$\pi R^2(\frac{2}{\pi})^{1/2}[1+\frac{3h}{2R}(1+\frac{\pi}{6})]$	$\pi R^4(\frac{1}{2\pi})^{1/2}[1+\frac{\pi}{4}+\frac{h}{R}+\frac{h^2}{2R^2}+\frac{h^3}{4R^3}(1+\frac{\pi}{6})]$

Two points are important here. The first is the magnitude of this term, which, for a given pressure and test mass dimension, is roughly three times larger than that obtained in a previous analysis [9]. Given the slow scaling with pressure, $S_F^{1/2} \sim p^{1/2}$, more ambitious gravitational wave missions, such as BBO [16] and DECIGO [17], which require lower acceleration noise, will have to reach much lower pressures than the 10^{-6} Pa value quoted here, or employ much larger test masses, with a larger mass more than offsetting the larger surface area in Eq. (19). In general, for given TM density, the resulting acceleration noise is given by $S_a^{1/2} \sim p^{1/2}s^{-2} \sim p^{1/2}M^{-2/3}$ for a TM of mass M and dimension s.

The second consideration concerns the applicability of this calculation to frequently encountered experimental conditions in which the test mass is surrounded by an enclosure which is only slightly larger than the test mass itself, or more generally is very close to the surrounding apparatus. This is certainly true for LISA, where the 46 mm cubic test mass is enclosed, with a gap of several mm, by the walls of a surrounding electrostatic position sensor [18]. This is also true for terrestrial gravitational wave observatories such as Advanced LIGO [19], where suspended TM are in close proximity to similar suspended masses used for actuation and thermal compensation. Similar physics of gas damping in a restricted geometry is fundamental to recent studies of "squeeze damping" in MEMS resonators [20].

For such cases, the assumption, used in the calculations presented here, of being able to calculate the force noise from the probability of a single impact – which is reasonable only if that impact is uncorrelated with previous and future impacts of the same molecule – collapses. In a tightly restricted geometry, a molecule emitted from a TM surface will, with high probability, strike the opposing enclosure wall only to then return to strike the same TM surface, imparting momentum with the same sign as the previous impact. Random walking, by molecular diffusion, from one side of the test mass to the other, will require many collisions and create a grouping of correlated collisions with force impulses with the same sign. This increases force noise, as the momentum imparted does not average out as quickly as for uncorrelated collisions in an infinite gas volume.

The random walk process down the narrow channels between test mass and enclosure is the same that determines the flow impedance in a tube. Given that test mass motion, inside a tight enclosure, will require gas flow around the test mass, this

impedance creates a pressure head, thus giving an alternate picture of the velocity-dependent force, or dissipation, that must accompany the increased force noise. These constrained volume effects are studied both experimentally and with numerical simulations, in a recently published parallel study [12], which, in addition to obtaining the force and torque noise formulas derived here in the limit of very large volume, demonstrates significantly increased damping in the case of gaps that are smaller than the TM itself.

References

[1] K. Numata, J. Horowitz, J. Camp, Phys. Lett. A 370 (2008) 91.
[2] C.A. Hagedorn, S. Schlamminger, J.H. Gundlach, in: Proc. 6th Int. LISA Symp., in: AIP Conf. Proc., vol. 873, New York, 2006, pp. 189–193.
[3] C.C. Speake, et al., Meas. Sci. Technol. 10 (1999) 508.
[4] L.D. Hinkle, B.R.F. Kendall, J. Vac. Sci. Technol. 8 (1990) 2802.
[5] L.D. Hinkle, B.R.F. Kendall, J. Vac. Sci. Technol. 10 (1992) 243.
[6] V.B. Braginsky, V.P. Mitropanov, V.I. Panov, Systems With Small Dissipation, University of Chicago Press, 1985.
[7] P.R. Saulson, Phys. Rev. D 42 (1990) 2437.
[8] R.G. Christian, Vacuum 16 (1966) 175.
[9] B.L. Schumaker, Class. Quantum Grav. 20 (2003) S239.
[10] P. Bender, et al., LISA: A cornerstone mission for the observation of gravitational waves, ESA-SCI (2000) 11, System and Technology Study Report.
[11] S. Vitale, et al., Nucl. Phys. B (Proc. Suppl.) 110 (2002) 209.
[12] A. Cavalleri, et al., Phys. Rev. Lett. 103 (2009) 140601.
[13] D.H. Davis, J. Appl. Phys. 31 (1960) 1169.
[14] B. Mercier, J. Vac. Sci. Technol. A 24 (2006) 529.
[15] S. Gruener, P. Huber, Phys. Rev. Lett. 100 (2008) 064502.
[16] J. Crowder, N.J. Cornish, Phys. Rev. D 72 (2005) 083005.
[17] S. Kawamura, Class. Quantum Grav. 23 (2006) S125.
[18] W.J. Weber, et al., Proc. SPIE Int. Soc. Opt. Eng. 4856 (2003) 43.
[19] Advanced LIGO Reference Design 2007, LIGO Tech. Rep. M060056, http://www.ligo.caltech.edu/docs/M/M060056-08/M060056-08.pdf.
[20] M.A.G. Suijlen, J.J. Koning, M.A.J. van Gils, H.C.W. Beijerinck, Sens. Actuators A 156 (2009) 171.